# Mathematics Notes/Ebook: A Comprehensive Guide for Secondary Schools

## Introduction

This ebook is designed to provide a clear and concise understanding of key mathematical concepts typically covered in secondary schools. It is structured into five main sections: Number and Numeration, Algebra, Geometry and Trigonometry, Statistics, and Revision. Each section builds upon foundational knowledge, progressively introducing more complex topics.

## SECTION B: ALGEBRA

This section introduces fundamental algebraic concepts, essential for solving equations and understanding relationships between quantities.

### 1. Polynomials

- **Definition:** An algebraic expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables.
- **Terms:** Monomial (one term), Binomial (two terms), Trinomial (three terms).
- **Degree of a Polynomial:** The highest power of the variable in the polynomial.
- **Types of Polynomials based on Degree:** Linear (degree 1), Quadratic (degree 2), Cubic (degree 3), etc.
- **Operations with Polynomials:**
    - **Addition and Subtraction:** Combine like terms.
    - **Multiplication:** Distributive property, FOIL method for binomials.
    - **Division:** Long division of polynomials, synthetic division (for linear divisors).
- **Factorization of Polynomials:**
    - Common factors.
    - Difference of two squares ($a^2 - b^2 = (a-b)(a+b)$).
    - Quadratic trinomials ($ax^2 + bx + c$).
    - Grouping method.
- **Roots of Polynomials:** Values of the variable that make the polynomial equal to zero.
- **Remainder Theorem:** If a polynomial P(x) is divided by (x-a), the remainder is P(a).
- **Factor Theorem:** (x-a) is a factor of P(x) if and only if P(a) = 0.

## 2. Variation

- **Definition:** Describes how one quantity changes in relation to another.
- **Types of Variation:**
    - **Direct Variation:** y = kx (y is directly proportional to x), where k is the constant of proportionality. As x increases, y increases proportionally.
    - **Inverse Variation:** y = k/x (y is inversely proportional to x). As x increases, y decreases proportionally.
    - **Joint Variation:** y = kxz (y varies jointly as x and z). Y is directly proportional to the product of two or more other variables.
    - **Partial Variation:** y = kx + c (y is partly constant and partly varies directly as x).
- **Solving Problems involving Variation:** Finding the constant of proportionality and using it to determine unknown values.

## 3. Inequalities

- **Definition:** Mathematical statements that compare two expressions using inequality signs (\<, , $leq$, $geq$).

- **Properties of Inequalities:**
    - Adding/subtracting a number from both sides does not change the inequality direction.
    - Multiplying/dividing by a positive number does not change the inequality direction.
    - Multiplying/dividing by a negative number reverses the inequality direction.
- **Solving Linear Inequalities:** Isolate the variable using inverse operations, remembering to reverse the inequality sign when multiplying or dividing by a negative number.
- **Solving Quadratic Inequalities:**
    - Find the critical points (roots of the corresponding quadratic equation).
    - Test intervals to determine where the inequality holds true.
    - Represent the solution on a number line or using interval notation.
- **Solving Simultaneous Linear Inequalities:** Find the intersection of the solution sets of individual inequalities.
- **Graphical Representation of Inequalities:** Shading the region that satisfies the inequality on a coordinate plane.

**1. Progression**

- Arithmetic Progression (AP):
  - **Definition:** A sequence of numbers where the difference between consecutive terms is constant. This constant difference is called the common difference (d).
  - **nth term:** $T_n = a + (n-1)d$, where 'a' is the first term and 'n' is the term number.
  - **Sum of n terms:** $S_n = n/2[2a + (n-1)d]$ or $S_n = n/2(a + L)$, where 'L' is the last term.
- Geometric Progression (GP):
  - **Definition:** A sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio (r).
  - **nth term:** $T_n = ar^{n-1}$, where 'a' is the first term and 'n' is the term number.
  - **Sum of n terms:** $S_n = a(1 - r^n)/(1 - r)$ (for r \< 1) or $S_n = a(r^n - 1)/(r - 1)$ (for $r$ 1).
  - **Sum to Infinity:** $S_{infty} = a/(1 - r)$ (for |r| \< 1).
- Problem Solving involving AP and GP.

## 2. Binary Operations

- **Definition:** A binary operation on a set combines two elements of the set to produce another element of the set. It is a rule that assigns to each ordered pair of elements from a set a unique element of that same set.
- **Examples:** Common binary operations are addition (+), subtraction (-), multiplication (×), and division (÷) on sets of numbers.
- **Properties of Binary Operations:**
    - **Closure:** A set is closed under a binary operation if applying the operation to any two elements of the set always produces an element within the same set.

- **Commutativity:** a * b = b * a (the order of operands does not affect the result).
- **Associativity:** (a * b) * c = a * (b * c) (the grouping of operands does not affect the result).
- **Identity Element:** An element 'e' such that a * e = e * a = a for all 'a' in the set.
- **Inverse Element:** For each element 'a', there exists an inverse $a^{-1}$ such that a * a^{-1} = a^{-1} * a = e (where 'e' is the identity element).

- **Solving problems involving defined binary operations.**

### 3. Matrices and Determinants

- **Matrices:**
  - **Definition:** A rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

    - **Order of a Matrix:** Number of rows $\square_{times}$ number of columns (e.g., $_{timesn}^{m}$).

  - **Types of Matrices:** Row matrix, column matrix, square matrix, zero matrix, identity matrix, diagonal matrix, scalar matrix, symmetric matrix.
  - **Operations on Matrices:**
    - **Addition and Subtraction:** Only possible for matrices of the same order. Add/subtract corresponding elements.
    - **Scalar Multiplication:** Multiply each element of the matrix by a scalar.
    - **Matrix Multiplication:** The number of columns in the first matrix must equal the number of rows in the second matrix.
    - **Transpose of a Matrix:** Rows become columns and columns become rows.
- **Determinants:**
  - **Definition:** A scalar value that can be computed from the elements of a square matrix.

    - **Determinant of a 2x2 Matrix:** For matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det(A) = ad - bc$.
    - **Determinant of a 3x3 Matrix:** Expansion by cofactors/minors.
    - **Singular Matrix:** A square matrix whose determinant is zero (it does not have an inverse).
- **Inverse of a Matrix:**
    - **Inverse of a 2x2 Matrix:** For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = (1/\det(A)) \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.
    - **Conditions for Inverse to Exist:** The determinant must not be zero.
- **Solving Simultaneous Linear Equations using Matrices (Matrix Inversion Method and Cramer's Rule).**