



Mathematics Notes/Ebook: A Comprehensive Guide for Secondary Schools

Introduction

This ebook is designed to provide a clear and concise understanding of key mathematical concepts typically covered in secondary schools. It is structured into five main sections: Number and Numeration, Algebra, Geometry and Trigonometry, Statistics, and Revision. Each section builds upon foundational knowledge, progressively introducing more complex topics.

SECTION A: NUMBER AND NUMERATION

This section focuses on essential numerical concepts, building a strong foundation for more advanced mathematical studies.

1. Number Bases

- **Definition:** A number base is the number of unique digits, including zero, used to represent numbers in a positional numeral system. The most common base is base 10 (decimal), which uses ten digits (0-9). Other common bases include base 2 (binary), base 8 (octal), and base 16 (hexadecimal).
- **Conversion from Other Bases to Base 10:** To convert a number from any base to base 10, multiply each digit by its corresponding positional value (power of the base) and sum the results.

- **Example:** Convert 1101₂ to base 10.

$$\begin{aligned}
 1101_2 &= (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\
 &= (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1) \\
 &= 8 + 4 + 0 + 1 = 13_{10}
 \end{aligned}$$

- **Example:** Convert 435₈ to base 10.

$$\begin{aligned}
 435_8 &= (4 \times 8^2) + (3 \times 8^1) + (5 \times 8^0) \\
 &= (4 \times 64) + (3 \times 8) + (5 \times 1) \\
 &= 256 + 24 + 5 = 285_{10}
 \end{aligned}$$

- **Conversion from Base 10 to Other Bases:** To convert a number from base 10 to another base, repeatedly divide the base 10 number by the new base and record the remainders. The new number is formed by reading the remainders from bottom to top.

- **Example: Convert 25₁₀ to base 2.**

$$\begin{array}{l} 25 \\ \text{div} 2 = 12 \end{array} \quad \text{remainder } 1$$

$$\begin{array}{l} 12 \\ \text{div} 2 = 6 \end{array} \quad \text{remainder } 0$$

$$\begin{array}{l} 6 \\ \text{div} 2 = 3 \end{array} \quad \text{remainder } 0$$

$$\begin{array}{l} 3 \\ \text{div} 2 = 1 \end{array} \quad \text{remainder } 1$$

$$\begin{array}{l} 1 \\ \text{div} 2 = 0 \end{array} \quad \text{remainder } 1$$

$$\text{So, } 25_{10} = 11001_2$$

- **Example: Convert 150₁₀ to base 8.**

$$\begin{array}{l} 150 \\ \text{div} 8 = 18 \end{array} \quad \text{remainder } 6$$

$$\begin{array}{l} 18 \\ \text{div} 8 = 2 \end{array} \quad \text{remainder } 2$$

$$\begin{array}{l} 2 \\ \text{div} 8 = 0 \end{array} \quad \text{remainder } 2$$

$$\text{So, } 150_{10} = 226_8$$

- **Arithmetic Operations in Other Bases:**

- **Addition:** Add digits column by column, carrying over when the sum exceeds the base.

- **Example:** Add 101_2 and 11_2 .

$$\begin{array}{r} 101_2 \\ + 011_2 \\ \hline 1000_2 \end{array}$$

($1+1 = 2 = 10_2$, so write 0 carry 1.
 $0+1+1$ (carry) = $2 = 10_2$, so write 0
 carry 1. $1+0+1$ (carry) = $2 = 10_2$, so
 write 0 carry 1.)

- **Subtraction:** Subtract digits column by column, borrowing from the left if a digit is smaller than the one being subtracted. Remember to add the base value when borrowing.

- **Example:** Subtract 11_2 from 101_2 .

$$\begin{array}{r} 101_2 \\ - 011_2 \\ \hline 010_2 \end{array}$$

($1-1 = 0$. $0-1$ (borrow from 1 in 2^2
 position becomes $2+0 = 2$). $2-1 = 1$.
 The 1 in 2^2 position becomes 0.)

- **Multiplication:** Similar to base 10 multiplication, but partial products are in the given base, and additions are also in that base.

- **Example:** Multiply 101_2 by 11_2 .

$$\begin{array}{r} 101_2 \\ \times 011_2 \\ \hline 101 \quad (101 * 1) \\ 1010 \quad (101 * 1, \text{ shifted}) \\ \hline 1111_2 \end{array}$$

- **Division:** Similar to long division in base 10.

2. Fraction, Decimals, Approximation and Percentage

- **Fractions:**
 - **Types:** Proper, improper, mixed numbers.
 - **Operations:** Addition, subtraction, multiplication, division of fractions.
 - **Simplification:** Reducing fractions to their lowest terms.
 - **Equivalent Fractions:** Fractions that represent the same value.
- **Decimals:**
 - **Place Value:** Understanding the value of each digit after the decimal point.
 - **Operations:** Addition, subtraction, multiplication, division of decimals.
 - **Converting Fractions to Decimals and Vice Versa:**
 - Fraction to Decimal: Divide the numerator by the denominator.
 - Decimal to Fraction: Write the decimal as a fraction with a power of 10 as the denominator, then simplify.
- **Approximation (Rounding and Significant Figures):**
 - **Rounding:** Shortening a number to a certain number of decimal places or whole numbers.
 - **Rules:** Look at the digit to the right of the desired position. If it's 5 or greater, round up the digit in the desired position. If it's less than 5, keep the digit as it is.
 - **Significant Figures:** The digits in a number that carry meaningful contribution to its measurement resolution.
 - **Rules:** Non-zero digits are always significant. Zeros between non-zero digits are significant. Leading zeros (0.005) are not significant. Trailing zeros are significant if the number contains a decimal point (e.g., 100.0 has 4 significant figures, 100 has 1 significant figure).
 - **Example:** Round 3.14159 to 3 decimal places: 3.142.
 - **Example:** State the number of significant figures in 0.00250: 3 significant figures.
- **Percentage:**
 - **Definition:** A fraction out of 100. (Symbol: %)
 - **Conversions:**
 - Percentage to Fraction: Divide by 100 and simplify. (e.g., 25% = $25/100 = 1/4$)
 - Percentage to Decimal: Divide by 100. (e.g., 25% = 0.25)
 - Fraction/Decimal to Percentage: Multiply by 100. (e.g., 0.75 = $0.75 \times 100\% = 75\%$)
 - **Calculations:**
 - Finding a percentage of a quantity.
 - Expressing one quantity as a percentage of another.
 - Percentage increase/decrease.
 - Profit and Loss.
 - Simple Interest.

3. Indices, Logarithm and Surds

- Indices (Exponents):

- Definition:** An index (or exponent) indicates the number of times a base number is multiplied by itself. (e.g.,

$$a^n = a$$

times a

times (n times)

dots

times a

- Laws of Indices:**

- Multiplication: $a^m \times a^n = a^{m+n}$

- Division: $a^m \div a^n = a^{m-n}$

- Power of a Power: $(a^m)^n = a^{mn}$

- Zero Index: $a^0 = 1$ (for $a \neq 0$)

- Negative Index: $a^{-n} = 1/a^n$
 $a^{m/n} =$

- Fractional Index: $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$

- Solving Equations involving Indices.**

- **Logarithm:**

- **Definition:** The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number. If $b^y = x$, then



$$\log_b x = y$$

- **Laws of Logarithms:**

-

Product Rule:



$$\log_b(MN) = \log_b M + \log_b N$$

-

Quotient Rule:



$$\log_b(M/N) = \log_b M - \log_b N$$

-

Power Rule: $\log_b(M^p) = p \log_b M$



-

Change of Base Rule:



$$\log_b x = \frac{\log_a x}{\log_a b}$$

-



$$\log_b b = 1$$

-



$$\log_b 1 = 0$$

- **Solving Logarithmic Equations.**
- **Common Logarithms (Base 10) and Natural Logarithms (Base e).**

- **Surds (Radicals):**
 - **Definition:** A surd is an irrational number that can be expressed as a root of an integer that cannot be simplified to a whole number. (e.g., $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ but not $\sqrt{4}$ as it simplifies to 2).
 - **Simplification of Surds:** Expressing surds in their simplest form. (e.g., $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$)
 - **Operations with Surds:** Addition, subtraction, multiplication, and division of surds.
 - **Rationalization of the Denominator:** Eliminating surds from the denominator of a fraction by multiplying both the numerator and denominator by the conjugate of the denominator (if it's a binomial surd) or the surd itself (if it's a monomial surd).

4. Sets

- **Definition:** A set is a well-defined collection of distinct objects. These objects are called elements or members of the set.
- **Notation:**
 - Roster method (listing elements): $A = \{1, 2, 3, 4\}$
 - Set-builder notation: $B = \{x \mid x \text{ is an even number less than } 10\}$
- **Types of Sets:**
 - Empty set (null set): $\square$ or $\{\}$
emptyset (contains no elements)
 - Finite set: Has a countable number of elements.
 - Infinite set: Has an uncountable number of elements.
 - Subset: A is a subset of B if all elements of A are also elements of B ($A \subseteq B$).
 - Proper subset: A is a proper subset of B if A is a subset of B and A is not equal to B ($A \subset B$).
 - Universal set ($\square$): The set of all elements under consideration.
mathcal{U}
 - Disjoint sets: Sets with no common elements.

- **Set Operations:**

- **Union ($\cup$):** The set of all elements that

are in A, or in B, or in both A and B. (A

$\cup$ B)

- **Intersection ($\cap$):** The set of all

elements that are common to both A

and B. (A $\cap$ B)

- **Complement (' or c):** The set of all elements in the universal set that are not in A. (A' or A^c)

- **Difference (-):** The set of all elements in A but not in B. ($A - B$ or $A \setminus B$)

- **Venn Diagrams:** Visual representation of sets and their relationships.

- **Cardinality of Sets:** The number of elements in a set, denoted as $n(A)$.

- **Problem Solving using Set Theory.**