Mathematics Notes/Ebook: A Comprehensive Guide for Secondary Schools

Introduction

This ebook is designed to provide a clear and concise understanding of key mathematical concepts typically covered in secondary schools. It is structured into five main sections: Number and Numeration, Algebra, Geometry and Trigonometry, Statistics, and Revision. Each section builds upon foundational knowledge, progressively introducing more complex topics.

SECTION C: GEOMETRY AND TRIGONOMETRY

This section delves into the study of shapes, sizes, relative positions of figures, and the properties of space, as well as the relationships between angles and sides of triangles.

1. Euclidean Geometry

- **Basic Concepts:** Points, lines, planes, angles (types: acute, obtuse, right, straight, reflex, complementary, supplementary), parallel and perpendicular lines.
- **Angle Properties:**
  - Angles on a straight line.
  - Angles at a point.
  - Vertically opposite angles.
  - Angles associated with parallel lines and a transversal (corresponding angles, alternate angles, interior/consecutive angles).
- **Polygons:**
  - **Definition:** Closed figures formed by straight line segments.
  - **Types:** Triangles, quadrilaterals, pentagons, hexagons, etc.
  - **Regular and Irregular Polygons.**
  - **Properties:** Sum of interior angles, sum of exterior angles, each interior/exterior angle of a regular polygon.
  - **Congruence and Similarity of Polygons:**
    - **Congruence:** Figures that have the same shape and size. (SSS, SAS, ASA, RHS for triangles)
    - **Similarity:** Figures that have the same shape but different sizes. (AAA, SSS, SAS for triangles)
    - **Ratio of Areas and Volumes for Similar Figures.**
- **Circles:**
  - **Basic Terms:** Radius, diameter, chord, arc, segment, sector, tangent, secant.

- **Circle Theorems:**
  - The angle at the centre is twice the angle at the circumference.
  - Angles in the same segment are equal.
  - The angle in a semicircle is a right angle.
  - Opposite angles of a cyclic quadrilateral sum to 180 degrees.
  - Tangent-radius property (tangent is perpendicular to the radius at the point of contact).
  - Angles between tangent and chord.
  - Intersecting chords theorem.
  - Intersecting secant theorem.

2. Mensuration

- **Definition:** The branch of mathematics that deals with the measurement of lengths, areas, and volumes of geometric figures.
- **Perimeter and Area of Plane Shapes:**
  - **Rectangles, Squares, Triangles:** Base, height, side lengths.
    - Parallelograms, Trapeziums, Rhombus, Kites.
    - Circles: Circumference ($C = 2pir$ or pid), Area ($A = pir^2$).
    - Sectors and Segments of a Circle: Arc length, area of sector, area of segment.
- **Surface Area and Volume of Solids:**
  - **Cubes and Cuboids:** Length, width, height.
  - **Cylinders:** Radius, height.
  - **Prisms:** Area of base, height.
  - **Pyramids:** Area of base, perpendicular height.
  - **Cones:** Radius, perpendicular height, slant height.
  - **Spheres and Hemispheres:** Radius.
- **Composite Shapes and Solids:** Calculating areas and volumes of figures made up of simpler shapes.

3. Loci

- **Definition:** The path traced by a point or a set of points satisfying a given condition.
- **Common Loci:**
    - Locus of points equidistant from a fixed point (a circle).
    - Locus of points equidistant from two fixed points (perpendicular bisector of the line segment joining the points).
    - Locus of points equidistant from a fixed straight line (a pair of parallel lines).
    - Locus of points equidistant from two intersecting straight lines (angle bisector).
- **Construction of Loci.**
- **Intersection of Loci:** Points that satisfy two or more locus conditions.

4. Coordinate Geometry

- **Definition:** The study of geometric figures using a coordinate system (Cartesian plane).
- **Cartesian Coordinates:** (x, y) representing a point's position.

- **Distance between Two Points:**
  $D =$
  $sqrt((x_2 - x_1)^2 + (y_2 - y_1)^2)$
- **Midpoint of a Line Segment:**
  $M = ((x_1 + x_2)/2, (y_1 + y_2)/2).$
- **Gradient (Slope) of a Line:**
  $m = (y_2 - y_1)/(x_2 - x_1).$
- **Equations of a Straight Line:**
  - Slope-intercept form: $y = mx + c$ (where c is the y-intercept).
  - Point-slope form: $y - y_1 = m(x - x_1).$
  - General form: $Ax + By + C = 0.$
- **Parallel Lines:** Have the same gradient $(m_1 = m_2)$.
- **Perpendicular Lines:** Product of their gradients is -1 $(m_1m_2 = -1)$.
- **Equation of a Circle:** $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.
- **Applications of Coordinate Geometry in Solving Geometric Problems.**

1. Trigonometry

- **Right-Angled Triangles:**
  - **SOH CAH TOA:**
    - Sine (sin): Opposite/Hypotenuse
    - Cosine (cos): Adjacent/Hypotenuse
    - Tangent (tan): Opposite/Adjacent
  - **Pythagoras Theorem:** $a^2 + b^2 = c^2$.
  - **Solving Right-Angled Triangles:** Finding unknown sides and angles.
- **Angles of Elevation and Depression:**
  - **Angle of Elevation:** Angle formed by the line of sight and the horizontal, when looking upwards.
  - **Angle of Depression:** Angle formed by the line of sight and the horizontal, when looking downwards.
- **Bearing:**
  - **Compass Bearing:** N, S, E, W and combinations (e.g., N30°E).
  - **Three-Figure Bearing:** Measured clockwise from North (e.g., 030°, 270°).

- **Sine Rule:** $a/sinA = b/sinB = c/sinC$. (Used for non-right angled triangles when you know two angles and a side, or two sides and a non-included angle).
- **Cosine Rule:** $a^2 = b^2 + c^2 - 2bc cosA$. (Used when you know two sides and the included angle, or all three sides).
- **Area of a Triangle:**
  - $A = 1/2ab sinC$ (when two sides and the included angle are known).
  - Hero's Formula: $A = sqrts(s-a)(s-b)(s-c)'$ where $s = (a+b+c)/2$ (semi-perimeter).
- **Trigonometric Ratios of Special Angles:** 0°, 30°, 45°, 60°, 90°.
- **Trigonometric Ratios of Angles Greater than 90°:**
  - Quadrants (CAST rule).
  - Reference angles.
- **Graphs of Trigonometric Functions:** $y = sinx'$, $y = cosx'$ $y = tanx$.
  - Amplitude, Period, Phase Shift.
- **Simple Trigonometric Equations.**